Portfolio Size – An Unrecognized Source of Risk

Robi Elnekave

President

Investment Management Institute, LLC
505 N Lakeshore Drive, Suite 2410
Chicago, IL 60611

Tel: (312) 923-0002
Email: robi@iminvestments.com

© 2001 All Rights Reserved
Abstract

The size of a portfolio, measured by the number of holdings, is a very important factor in active management. The number of holdings in a portfolio plays a significant role in whether or not the excess return of the strategy is captured. In addition, portfolio size has a considerable impact on the variability of the outcomes; that is whether or not the tracking error between the portfolio and the underlying strategy is minimized. Yet the number of positions held in a portfolio is seldom the topic of a deliberate study by active managers.

In this article we will analyze the questions related to ideal portfolio size from three perspectives: as a statistical sampling problem, as the outcome of the fundamental law of investing and as the result of iterative simulations. In the process we hope to not only demonstrate the critical importance of actively selecting the number of holdings for a given strategy but also introduce a set of tools for this purpose.
Introduction

Many decisions play a role in determining the ultimate performance of an active manager. Among the most widely recognized contributors to investment performance are: the type of investment strategy utilized, otherwise known as the manager’s style; the predictive strength of the investment models, otherwise recognized as the manager’s skill; and the portfolio trading conventions, or trading efficiency. Needless to say, there are numerous other decisions that play a role in attempting to achieve the established performance targets. However, there is one critical aspect of portfolio management that is seldom taken into consideration.

In active management, the size of the portfolio, defined by the number of positions held, is rarely if ever studied and analytically selected. Even when a conscious decision plays a role in determining the size of the portfolio, risk considerations seldom enter into the decision process. Portfolio size is either based on the ability of the manager to keep track of his or her holdings, or adjusted to accommodate the dollars under management, or tempered by the degree of perceived burden on back office operations.

This article will show that portfolio size has a demonstrable influence on investment performance. First, the number of holdings in a portfolio plays a significant role in determining how effectively the performance of the underlying strategy is being captured. Second, portfolio size has a considerable influence on the variability of outcomes. Finally, portfolio size has the potential to reduce market impact, and therefore transaction costs, increasing trading efficiency.
We will investigate the problem of determining optimum portfolio size from three perspectives: as a statistical sampling problem, as an outcome of the fundamental law of investing introduced by Grinold and Kahn, and as the conclusion of iterative simulations. In the process, we hope to convince the reader to recognize portfolio size as an important factor in achieving performance targets. We will also introduce a set of quantitative tools to help the reader analytically determine portfolio size.

**Portfolio Size as a Statistical Sampling Problem**

The challenge of an investment manager, charged with constructing a new stock portfolio or managing an existing one, is to capture the information content of the underlying strategy. However, as is usually the case, the underlying strategy provides more top rated choices than needed. An inherent trait of every investment strategy is that some of these choices are winners and some are laggards. This is due to the imperfect information content of the strategy. Therefore, a question that might be asked is how many choices should be made in order to realize the mean population return. In other words, the top picks offered by the strategy have a certain mean return. At a minimum, the investment manager, lacking any other source of information, should capture this mean return with a high likelihood. One way to accomplish this is via statistical sampling techniques. Through these techniques, the manager selects a representative sample that ideally has the same percentage of winners as the population. The percentage of winners within a group is also called the hit ratio of the strategy. The rationale behind
this approach is, as the sample hit ratio approaches that of the top picks, the sample mean return will also come close to the population mean.

We recognize that determining sample size via statistical techniques is a naïve way to approach this problem. First, sampling techniques do not take into account domain specific attributes such as the interrelationship of the potential buy candidates. Second, this approach assumes a normal distribution. Stock returns, on the other hand, are only approximately normal, typically containing fat tails with positive skewness. However, an unencumbered, simple view of the problem is a good first step before delving into the realm of more complex and customized solutions.

From a statistical perspective, sample size is a function of the hit ratio of the investment strategy, the standard deviation of the hit ratio, the confidence level selected, and the acceptable range of error band around the hit ratio (See Kachigan 1994). In mathematical terms the equation is:

\[
N = \frac{(z_\alpha - z_\beta)^2 \alpha^2}{(\mu_1 - \mu_0)^2}
\]

(1)

N: Sample size

\(z_\alpha\): Confidence level expressed as number of standard deviations

\(z_\beta\): Power of a test expressed as number of standard deviations (i.e. probability of correctly rejecting a false hypothesis)

\(\alpha\): Standard deviation of the hit ratio

\(\mu_0\): Population hit ratio

\(\mu_1\): Desired hit ratio
An example might help clarify this formula. Let’s assume we have an investment strategy with a 55% hit ratio. In other words, 55% of the top picks identified by the strategy exceed the population mean. From this group of stocks we would like to create a portfolio that would ideally match the hit ratio of the population with a +/- 2% error band and 99% confidence level. If we input these parameters into equation I the sample size comes out as 225 stocks. Therefore, according to the statistical sampling formula, with a 225 stock portfolio, we would with 99% confidence be within 2% of the hit ratio of the strategy, thereby maximizing the probability of realizing the mean excess return.

Due to the generic nature of this statistical technique, there are a few problems with this approach. First, the formula does not take into account the population size to predict sample size\(^1\). In the example used, we found a 225 stock portfolio to be optimal for capturing the hit ratio. However, due to the buy and sell rules utilized, the manager might not be able to invest in 225 names. Second, the formula does not take into account the characteristics of the domain; it purely views the problem as a statistical sampling problem. In the realm of sampling, every new candidate is an independent observation unrelated to all others. However, in portfolio management the interrelationship of holdings is an important consideration in risk reduction. Though, in spite of its shortcomings, the statistical technique is effective in demonstrating the critical interrelationship between hit ratio and sample size. Since investment disciplines by their very nature incorporate limited information content and by default have low hit ratios, the
conclusion that large portfolios are needed to capture information content is an accurate one.

**Implications of the Fundamental Law of Investing on Portfolio Size**

In the previous section, we explored using a statistical sampling technique to determine the optimal size of a portfolio. Since this technique is equally applicable to a large set of problems, it was not able to take into account some domain specific information that makes a difference in the answer we get. Here we will explore the use of a more specific technique to infer the optimal portfolio size.

Grinold & Kahn, in their book Active Portfolio Management, introduce a mathematical relationship they term “The Fundamental Law of Active Management.” This law interrelates an investment manager’s opportunity with the manager’s skill and breadth. The investment manager’s skill is defined by the information coefficient (IC). The IC measures the correlation between the prediction and the outcome. In other words, the IC captures the degree of a manager’s forecast accuracy. For the purposes of this law, breadth (BR) is defined as the number of independent decisions a manager makes. For instance, if a manager makes buy sell decisions on 150 stocks a year, assuming each decision is independent of the others, we can say this manager has a breadth of 150. The opportunity set is defined by the information ratio (IR). The IR is calculated by dividing excess return ($\alpha$), that is, return in excess of a benchmark, by residual risk ($\omega$), defined by the volatility of the excess return time series. The IR is indicative of a manager’s achievement (ex-post) or opportunity (ex-ante).
In functional form the fundamental law of active management is expressed as follows:

\[ IR \cong IC \sqrt[2]{BR} \quad (II) \]

This law, in its original form, is effective in telling us the number of active bets (BR) required to realize a certain information ratio (IR) given a skill level (IC). However, the number of active bets, that is, the number of independent, active decisions we make per year, does not tell us anything about portfolio size. But there is one fact that can help us make the leap between the number of bets and portfolio size, namely, that the manager makes decisions within the context of the portfolio. This realization does not imply that we end up buying every security we analyze; it just means we make decisions to buy or not in the context of our holdings. With the help of this insight, we can modify the formula and relate it to portfolio size. In order to draw conclusions we need to expand BR and re-arrange the formula. For our purposes, we will define BR as a function of active decisions for the portfolio holdings (N), the frequency of these decisions (F), the annual turnover rate of the portfolio (T), and the number of forecasts per trade (\( \lambda \)).

\[ BR = (N \times F) + (N \times T \times \lambda) \quad (III) \]

A more intuitive way to look at this formula is to interpret the first set of terms between parentheses as active forecasts on existing positions, and the second set of terms between parentheses as active forecasts for all new positions considered, whether they end up
being included in the portfolio or not. Substituting the expanded form of BR and solving the fundamental law for \( N \) gives us:

\[
N = \frac{\left( IR^2 / IC^2 \right)}{(F + T^* \lambda)} \quad (IV)
\]

This law provides us with insights about the trade-offs we can make, but it is not an operational tool. The most restrictive assumption of the law is that each decision that goes into BR be independent. However, in spite of this the formula is a very powerful guide.

For instance, a manager with a given skill level (IC) who wants to achieve a higher information ratio (IR) has a number of choices. Provided the skill across bets can be kept constant, the manager can expand the size of the portfolio or alternatively, the manager can keep the portfolio size constant but increase turnover to reach the same result. However, from a practical perspective, the first option is more favorable since increasing turnover also creates drag on the portfolio performance in the form of taxes and transaction costs. Figure I below helps to illustrate this point pictorially.

<<Insert Figure I Here>>
A more informative and relevant exercise might be to use this formula to evaluate the decisions of a hypothetical fund manager. If this hypothetical manager has an IC of 0.05, makes annual forecasts, typically holds 40 positions, turns over the portfolio every 24 months (i.e. 50% annual turnover), and on average considers 2 stocks for every purchase, this manager can hope to achieve an IR of 0.45 – not a very satisfactory result.

Let’s look at this same manager from a different, and more intuitive perspective. For illustration purposes, let us assume that this hypothetical manager is talented and on average is able to generate 2% excess return. Since IR is comprised of expected excess return \( \alpha \) over variability of expected excess return \( \omega \) -- defined by standard deviation -- we will solve the fundamental law formula for variability using the above inputs.

\[
\omega \approx \frac{\alpha}{IC \sqrt{(N \times F) + (N \times T \times \lambda)}}
\]

According to the fundamental law, the manager with the above characteristics will have 4.5% variability around his/her performance. In other words, even though this manager has talent in picking stocks he/she would underperform 34% of the time purely because of constrained portfolio size.

This brings us to the second point of this article, which is the influence of holdings on variability of performance. Using the parameters for the hypothetical manager (\( \alpha = 2\% \), IC = 5% and turnover = 50%) and equation V, we varied the number of portfolio
holdings. The chart below highlights the impact of holdings on variability of alpha while keeping all else constant.

<<Insert Figure II here>>

According to equation V, a manager who holds 200 positions rather than the conventional 80 can reduce the standard deviation of the portfolio excess returns by 37% and in the process raise the information ratio from 0.7 to a healthy 1.2. This manager would also significantly reduce the likelihood of under performing and that of getting fired.

The equations introduced by Grinold & Khan bring us still closer to understanding the influence of portfolio size on capturing the information content of a strategy and managing variability. In the next section we will explore this problem from a practical perspective.

**Optimal Portfolio Size Through Iterative Simulations**

The ultimate question at this point is, whether or not real world evidence supports the insights we have gained from the theoretical framework? In order to answer this question from a practical perspective we conducted portfolio simulations for three investment styles: value, growth and momentum, for a period of 12 years, starting January 1987 and ending December 1999. The simulations were based on a historical database of rankings
provided by Chicago Investment Analytics (CIA). CIA is an institutional quantitative investment research group within Charles Schwab & Company Inc. This group’s mission is to investigate market inefficiencies and to capture these inefficiencies in stylistic stock selection models. The database provided by CIA contained 4472 companies for which at any point in time we had rankings for all three investment styles tested. All three stylistic models were quantitatively formulated and provided stock rankings that ranged from 1, which is considered a favorable rating, to 100, which is considered a poor rating. According to this ranking scheme the ordinal value of the ranking is indicative of expected price performance.

Using the value, growth and momentum rankings we formulated buy and sell rules. For the purposes of this simulation, we chose rankings 50 or greater as the sell point and rankings 10 or less as the buy point. Then using the buy candidates we first created and then traded portfolios of 50, 150 and 250 stocks for a 12 year time horizon on a monthly basis. However, rather than making one pass through this process, which is fairly conventional, we made 150 passes for each and every portfolio size. At each rebalance point, all stocks whose rankings were 50 or greater were sold from the portfolio. As the buy rule almost always gave us more choices than needed, after eliminating existing holdings from consideration, we randomly selected the equivalent number of names to replace the stocks which were sold. Repeating this random selection process 150 times over a 12 year horizon provided us with a distribution of compounded returns for each portfolio size and each style simulated. In order to keep the examples simple the simulations did not take into account transaction costs and market impact. The reader
needs to keep in mind that portfolios with fewer names are likely to have more market impact as larger dollar allocations are invested in each position. The three tables presented below summarize our findings.

<<Insert Table I Here>>

<<Insert Table II Here>>

<<Insert Table III Here>>

As expected, the simulations confirm many of the theoretical findings; however, they also reveal some new insights.

The average excess return figures for all three portfolio sizes are similar across the three stylistic portfolios. However, there is a slight upward trend for the larger portfolios. We claim that this is due to two factors. First, larger sample sizes capture the population average more accurately. This outcome is an artifact of the law of large numbers; that is, as the sample size increases, the mean of the population approaches that of the true mean. Second, since returns are skewed the larger portfolios are more likely to have more of the stocks with large excess than smaller portfolios. The simulation results were supportive of this point. Each simulation for a given style and given portfolio size entailed creating 23,250 portfolios\(^2\). When we compared the performance of the 50 stock portfolios to the performance of the 250 stock portfolios across the 23,250 instances, we found that the 250 stock portfolios exceeded their 50 stock counterpart 53% of the time, hence the slight upward bias in average returns.
The standard deviation of the mean excess return of the larger portfolio samples, as predicted by Grinold & Kahn, is also lower. The standard deviation of the 250 stock portfolio sample is approximately 1/3 of the 50 stock portfolio counterpart. The Fundamental Law of Investing successfully predicted the direction of this relationship; however, the magnitude of risk diversification seems to be much higher in real life portfolios\(^3\). This might be due to the fact that Grinold & Khan do not take into account the influence of the interrelationship of holdings.

Also, as implied by the lower standard deviation figures, the larger portfolios have lower maximum returns and higher minimum returns. In other words, the managers with larger portfolios are much less likely to disappoint; however, they are also less likely to surprise on the upside. As a result, the larger portfolios better capture the alpha of the strategy being utilized.

Finally, it does not appear that large portfolio sizes are beneficial to one style of investing versus another. All three strategies tested, even though they have very different investment characteristics, benefited very similarly from expanded portfolio sizes.

**Conclusion**

Studies conducted in the 70’s by Fama (See Fama 1976) demonstrated that by the time a portfolio includes 15-20 stocks, 95% of the variability of that portfolio is eliminated. The conclusions of this 25 year old study have been misinterpreted by both the general public
and the investment community. In this paper, we have demonstrated that conventional portfolio construction techniques, where fewer rather than more stocks are held, result in significant variability in returns across portfolios created with the same trading rules.

In addition, contrary to popular wisdom, we have demonstrated that a large portfolio does not result in sacrifice of alpha. Quite to the contrary, it helps capture the information content embedded in the strategy much more effectively.

A further benefit of large portfolios is that they are more efficient to trade and allow for growth of assets under management. After all, keeping all else equal, which manager is more likely to have lower transaction costs: the one who holds 50 positions and manages $1B or the one who holds 250 positions?

Consistency of performance is another benefit of large portfolios. In this day where managers are expected to track their benchmark while adding value, consistency is highly rewarded. Neither institutional investors nor the general public have the patience to let the manager prove his/her talent over an extended period of time⁴. Managers who disappoint are quickly replaced.

An instance where a smaller portfolio can be justified is if the manager has the talent to somehow sub-select from among the purchase candidates of the primary strategy. Under these conditions, the simulations clearly demonstrate the benefit of holding small, concentrated portfolios. As confirmed by the simulations, with a smaller portfolio a
manager can achieve a higher maximum return. However, we would like to caution the courageous manager that this is akin to climbing Everest: only a few succeed, while the rest just keep trying.

We would like to leave the reader with one last message to consider. Make portfolio size a conscious, analytical decision based on the Information Ratio selected, and not an outcome of back office limitations or haphazard choice.
Figure 1
Impact of Skill on # of Portfolio Decisions

Figure II
Standard Deviation as a Function of Portfolio Size
(Alpha = 2%, IC = 5%, T/O = 50%)
Table I. Value portfolio simulations from 12/87 – 12/99  
(150 monthly iterations)

<table>
<thead>
<tr>
<th>Number of Stocks in the portfolio</th>
<th>50</th>
<th>150</th>
<th>250</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average exc. return</td>
<td>4.27</td>
<td>4.67</td>
<td>5.07</td>
</tr>
<tr>
<td>Minimum return</td>
<td>-1.03</td>
<td>2.47</td>
<td>3.87</td>
</tr>
<tr>
<td>Maximum return</td>
<td>8.77</td>
<td>6.47</td>
<td>6.47</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>1.6</td>
<td>0.7</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Table II. Growth portfolio simulations from 12/87 – 12/99  
(150 monthly iterations)

<table>
<thead>
<tr>
<th>Number of Stocks in the portfolio</th>
<th>50</th>
<th>150</th>
<th>250</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average exc. return</td>
<td>3.17</td>
<td>3.07</td>
<td>3.17</td>
</tr>
<tr>
<td>Minimum return</td>
<td>-0.03</td>
<td>1.07</td>
<td>2.07</td>
</tr>
<tr>
<td>Maximum return</td>
<td>8.47</td>
<td>4.67</td>
<td>4.47</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>1.5</td>
<td>0.7</td>
<td>0.4</td>
</tr>
</tbody>
</table>

Table III. Momentum portfolio simulations from 12/87 – 12/99  
(150 monthly iterations)

<table>
<thead>
<tr>
<th>Number of Stocks in the portfolio</th>
<th>50</th>
<th>150</th>
<th>250</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average exc. return</td>
<td>8.87</td>
<td>9.37</td>
<td>9.57</td>
</tr>
<tr>
<td>Minimum return</td>
<td>4.87</td>
<td>7.57</td>
<td>7.97</td>
</tr>
<tr>
<td>Maximum return</td>
<td>13.77</td>
<td>11.27</td>
<td>10.87</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>1.7</td>
<td>0.8</td>
<td>0.6</td>
</tr>
</tbody>
</table>
Endnotes
1. Population size is typically an optional input into most sampling formulas. This is due to the fact that the mathematics of probability makes population an irrelevant input. The only exception to this rule is if sample size exceeds the population size; then the formula is modified to take the population size into account.

2. The study covered 155 time periods and for each time period 150 iterations were conducted resulting in 23250 portfolios.

3. According to formula V a portfolio that has 250 names, turns over 100% a year, generates 2% alpha, with an IC of 0.09 should, on average, produce a standard deviation of 1.4%.

4. Scott Steward presents an outstanding paper on the importance of manager selection and consistency of returns.

References
Kachigan, Statistical Analysis - An Interdisciplinary Introduction to Univariate & Multivariate Methods, 1994 Radius Press, pp 189-191

Grinold & Kahn. Active Portfolio Management, 1995 Probus Publishing


Fama, Eugene F., Foundations of Finance, Basic Books 1976